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First, we will briefly describe our recently submitted, accepted, or published results. Then afterwards, we shall delineate our current and future works.

1. **Conditions for Stationary Pulse Propagation in the Strong Dispersion Management Regime**, T. I. Lakoba, J.Yang, D. J. Kaup and B.A.Malomed (Accepted by Optics Letters). This is a very important paper, in that it gives the first variational solution of a key problem in fiber optics communication. It has been recently shown that by using fibers of different dispersions, one could very stably, propagate solitary wave pulses over long distances and at high rates. This paper verifies those results variational, and also predicts new and interesting regimes, in which this can be used. In this paper, we have also verified these results numerically.

2. **Soliton Pulse Compression in the Theory of Optical Parametric Amplification**, with E. Ibragimov & Allan Struthers (accepted by Optics Comm.). With these other authors, we have determined that the upconversion process of the three-wave resonant interaction (3WRI) does successfully explain certain recent experiments in pulse compression and frequency conversion. The importance of this is that this process allows one to generate high intensity optical pulses, of arbitrary frequency, by using a high intensity, fixed frequency pump, and a variable frequency, but low intensity idler. The frequency of the signal pulse will be at the difference of the frequencies of the pump and idler. We show how to determine the necessary pulse parameters to obtain various signal outputs.

3. **Asymptotic Behavior of N–Soliton Trains of the Nonlinear Schrödinger Equation**, V. S. Gerdjikov, D. J. Kaup, I. M. Uzunov and E. G. Evstatiev, Phys. Rev. Lett. **77**, 3943-6 (1996). Consider a train of solitons, all of almost equal amplitudes and velocities, and all well separated from one another. In this case, one can determine the evolution of these solitons by simply evaluating the nonlinear interaction due to the small overlaps of the tails. When one does this, he can reduce the equations for the pulse centers and amplitudes to a set of ODE's, which are now a set of lattice equations, which then can be identified with a complex Toda lattice, which is again integrable. Thus one can now assume various initial arrangements of soliton pulses, then use the integrable complex Toda lattice equations to determine the asymptotics.

Using these asymptotics, one could then identify what initial phasing and amplitude variations would be the most stable and/or unstable.

4. **Stability and Quasi-Equidistant Propagation of NLS Soliton Trains**, V. S. Gerdjikov, E. G. Evstatiev, D. J. Kaup, G. L. Diankov, and I. M. Uzunov (accepted by Phys. Letters A) This is a more detailed analysis of the above paper.

5. **Criterion and Regions of Stability for Quasi-Equidistant Soliton Trains**, V. S. Gerdjikov, E. G. Evstatiev, D. J. Kaup, G. L. Diankov, and I. M. Uzunov (To appear in the Proceeding of the 1997 NEEDS workshop, Crete, 22 June, 1997.) See above work.

6. **The N-Soliton Interactions, Complex Toda Chain and Stable Propagation of NLS Soliton Trains**, V. S. Gerdjikov, E. G. Evstatiev, D. J. Kaup, G. L. Diankov, and I. M. Uzunov. (To appear in the Proceedings of *Advanced Photonics with Second-Order Optically Nonlinear Processes*, NATO Advanced Study Institute, Sozopol, Bulgaria, 25 Sept., 1997.) Prof. Gerdjikov was given a special invitation to attend this NATO Institute and present a lecture and a poster on our joint, above work.

7. **On the Inverse Scattering Transform for the Benjamin-Ono Equation**, D. J. Kaup and Y. Matsuno (accepted by Stud. Appl. Math.). Prof. Matsuno visited Clarkson University for 10 months. During that time, we made tremendous progress in understanding how to extend and expand the inverse scattering transform (IST) for the Benjamin-Ono (BO) equation and its soliton solutions for deep water. We found that the extant version of this IST was incomplete. Upon the completion of this work, we have been able to make large strides in completing the full theory of the BO equation. (see below).

8. **Linear Stability of Multiple Internal Solitary Waves in Fluids of Great Depth**, Y. Matsuno and D. J. Kaup, Phys. Lett. A **228**, 176-181 (1997). Here we show that N-soliton solutions are stable and we also formulate a perturbation theory for soliton interactions in deep water.

9. **Initial Value Problem of the Linearized Benjamin-Ono Equation and Its Applications**, Y. Matsuno and D.J. Kaup, J. Math. Phys. **38**, 5198-224 (1997). Here we consider the radiation part of the solution of the BO equation, and detail its properties.

10. **Complete Integrability of the Benjamin-Ono Equation by Means of Action-Angle Variables**, D.J. Kaup, T.I. Lakoba and Y Matsuno (Accepted by Physics

Letters A). Based on our earlier works on the BO, we now can show that this equation is completely integrable, and and detail the action-angle variables, and their properties.

11. **Perturbation Theory for the Benjamin–Ono Equation**, with T.I. Lakoba and Y. Matsuno (Submitted to Physica D) Here is the final, complete work on the BO equation. This details the perturbation theory of the BO equation, for any inital data and for any (small) perturbation.

12. **Perturbation Theory for the Manakov Soliton and Its Application to Pulse Propagation in Randomly Birefringent Fibers**, D. J. Kaup and T. I. Lakoba, Phys. Rev. E **56**, 6147-65 (1997). For solitons in optical fibers, the correct equation to use is the Manakov system, with perturbations. Here we detail the perturbation theory for this important equation, and give the closure relation. We then use the theory to evaluate the effects of random changes in the polarization and phase velocities on soliton propagation. We find that a vector soliton, such as the Manakov soliton, is more stable than the scalar form.

13. **Asymmetric Solitons in Mismatched Dual-Core Optical Fibers**, D.J. Kaup, T.I. Lakoba and Boris A. Malomed JOSA B **14**, 1199-1206 (1997). This was an ambitious variational calculation to test if the variational method could predict new solitary waves. We used the method to determine possible solitary wave solutions for a Dual-Core fiber with an arbitrary mismatch between the cores. We then numerically verified, with exact numerical solutions, several of these solitary waves. This has validated this method, and also revealed a new type of a continuous-solitary wave, best described as a solitary wave setting on top of a harmonic wave, and which we termed as a "delocalized solitary wave". Standard numerical solutions would not have been able to find these solutions. Although they are usually unstable, we suspect that they may be important and useful as transient states in a coupler.

14. **Solitons in Nonlinear Fiber Couplers with two Orthogonal Polarizations**, T. I. Lakoba, D. J. Kaup and Boris A. Malomed, Phys. Rev. E **55**, 6107-20 (1997). This is mostly the work of Dr. Lakoba, who successfully applied the variational method to a complex, four-component system. He has delineated the possible solitary waves, their probable stability, and interpreted the structure of the solitary waves as different parameters are changed.

15. **Stability of Solitons in Nonlinear Fiber Couplers with two Orthogonal Polarizations**, T. I. Lakoba and D. J. Kaup, Phys. Rev. E **56**, 4791-4802 (1997). This details the stability theory in the above work.

16. **Solitary Waves in Perturbed Generalized Nonlinear Schrödinger Equations**, with Jianke Yang (submitted to J. Math. Phys.) Frequently the nonlinearity can be more general than the Manakov model. In this case, the system is no longer integrable, and more general techniques are required. Here we study a very general case of two coupled NLS equations, and detail a singular perturbation theory for the system. We also illustrate how one can determine the number of bound states, and whether or not these bound states are stable.

17. **Gap Solitons in Asymmetric Dual–Core Nonlinear Optical Fibers**, with B.A. Malomed (submitted to JOSA B). Here we mainly study a new class of solitary wave, which occurs when the dispersion in one core is the opposite of another. In one core, the dispersion is normal, and in the other, it is anomalous. Thus one core, by itself, cannot support solitons, while the other one can. However, we find that the solitary waves in this dual core fiber can have more than 50% of its energy stored in the *normal* core. There are also new solitary wave states, where the tails decay exponentially, but with oscillations.

18. **Density Profiles and Current Flow in a Crossed–Field Amplifier**, D.J. Kaup and Gary E. Thomas, J. Plasma Phys. **58**, 145-61 (1997). Here we discuss how the electron density profile inside of standard CFA's give rise to a DC current density, which can be theoretically calculated.

19. **Stationary Operating Density Profiles in a Crossed–Field Amplifier**, D.J. Kaup and Gary E. Thomas (accepted by J. Plasma Phys.). There is an old classical problem of how to calculate the electron density profile for an operating magnetron or crossed-field amplifier (CFA). Here we show that one can numerically solve the nonlinearly coupled equations for the density profile and the RF waves, and then numerically determine the stable electron density profile as well as the resulting current density. Numerical data agrees reasonably well with experiments. Figures are shown of the composite solution for the density, which show distinct spokes, well known from particle simulations.

20. **Relativistic Density Profiles and Current Flow in a Crossed-Field Relativistic Electron Vacuum Device**, D.J. Kaup, T.I. Lakoba, and Gary E. Thomas, Proceedings of the 1997 SPIE Conference, Intense Microwave Pulses, Section V, 31 July-1 August, San Diego, CA, Vol. 3158, pp. 137-44 (1997). The plasma theory for the nonrelativistic case (see above) is extended into the relativistic regime.

21. **Solutions of Degenerate Two–Photon Propagation from Bäcklund Transformations**, H. Steudel, R. Meinel and D.J. Kaup, J. Mod. Optics **44**, 287-303 (1997). With femtosecond pulses becoming a reality, the feasibility of using various four–wave interactions is becoming practical. One of these four–wave interactions is the process known as "two–photon propagation" (TPP). There is a degenerate case of TPP where the pump and the decay wave are identified, causing the reaction to occur very rapidly, which is the reason for the experimental interest in this particular process. This paper shows that this process can generate unlimited pulse compression when there is only amplitude modulation. It also shows how to solve these equations in general, as long as there is no phase variation (frequency shift) across the pulse. For the interaction described above, there also exist Bäcklund transformations for this problem, since these equations are integrable. Here we detail various solutions obtainable by the Bäcklund transformation.

Work in progress.

1. **The Inverse Scattering Transform for Degenerate W–Problems, D.J. Kaup and H. Steudel.** (in preparation). Here we look at solving the degenerate TPP equations when there is a phase variation across the pulse. We are finding that any phase variation will limit the pulse compression. However, the complexity of this problem has encouraged us to study a much simplier, but related system, simple harmonic generation with walk-off.

2. **Rigorous Theory of Optical Pulse Shape in Dispersion Management** (with T.I Lakoba) Here we are replacing the previous variational results with exact analytical calculations based on Hermite polynomials. We find that only 4 orders are necessary to reproduce the exact numerical results.

3. **Novel Non-Gap Solitary Waves Created by Harmonic Generation in a Dual Core Fiber** (with Boris Malomed and J. Yang) Here we show that when one uses a pulse and its harmonic in a dual core fiber, then one can have a solitary wave that *coexists with the continuous spectra.* This is a remarkably new type of solitary wave, and one never before obtained. If it is stable, then this will be a very noteworthy result.

4. **Efficient Pulse Compression by Backscattering**, D.J. Kaup, A. Struthers and E. Ibragimov, (in preparation). With these authors, we have found that the backscattering process can be just as effective as the upconversion process, in producing pulse compression and frequency conversion.

5. **Operating Range of the VMS-1873 Magnetron – Comparison of Theory and Experiment**. (in preparation) We are applying the relativistic theory to this device to have a comparison with the analytics.

6. **Operating Density Profiles for Relativistic Magnetrons**, (in preparation).